PROBLEM OF DETERMINING THE LEWIS NUMBER

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Theoretical relations are derived for determining the Lewis number with the superheat of water vapor taken into account.

Calculations of heat and mass transfer from air or the products of combustion of natural gas to water in contact-type heat exchangers are based on the validity of the Lewis number [1-3]

$$\frac{\alpha_c}{\beta_x} = c_g + c_v X_g, \tag{1}$$

which follows from the condition

$$\frac{dt_g}{dX_g} \approx \frac{t_g - t_\omega}{X_g - X_\omega}. \tag{2}$$

This approximation does not, however, take into account the superheat of water vapor and using it for calculations leads to errors in the calculation of the Lewis number. The object of this study here is to establish the magnitude of these errors, depending on the conditions under which heat and mass transfer in contact-type heat exchangers occur.

On the I—d diagrams the X-scale is uniform and the $X_g = \text{idem}$ lines are parallel to the ordinate axis. At the same time, the $t_g = \text{idem}$ lines (isotherms) taking into account the superheat of water vapor constitute a family of divergent straight lines with the variable slope $dt_g/di_g = 597 + 0.45X_g$. Consequently, the quantity dt_g/dX_g representing the temperature increment in the gas stream at temperature t_g is also variable in the direction of the process beam $\varepsilon = (i_g - i_\omega)/(X_g - X_\omega)$, the latter characterizing the process conditions of heat and mass transfer at a surface element of the contact-type heat exchanger. The quantity determined from expression (2), on the other hand, represents the mean temperature increment in the gas stream $(dt_g/dX_g)_m$ due to a change in the gas temperature from t_g to t_ω with $\varepsilon = \text{const}$, i.e., when the gas stream has reached full equilibrium and the water temperature remains constant. Accordingly, Eq. (2) should be written as

$$\left(\frac{dt_g}{dX_g}\right)_m = \frac{t_g - t_\omega}{X_g - X_\omega}. \tag{3}$$

In view of these considerations, the well-known equation for the Lewis number can be rewritten as

$$\frac{\alpha_g}{\beta_x} \frac{t_g - t_\omega}{r(X_g - X_\omega)} = \frac{c_g + c_v X_g}{r} \frac{dt_g}{dX_g} K. \tag{4}$$

The correction factor in Eq. (4) is

$$K = \left(\frac{dt_g}{dX_g}\right)_m \Big/ \frac{dt_g}{dX_g}. \tag{5}$$

With the superheat of water vapor taken into account, the Lewis number is then

$$\frac{\alpha_c}{\beta_x} = (c_g + c_v X_g) K. \tag{6}$$

For calculating the quantity dt_g/dX_g needed for determining the factor K (5), we consider the expression

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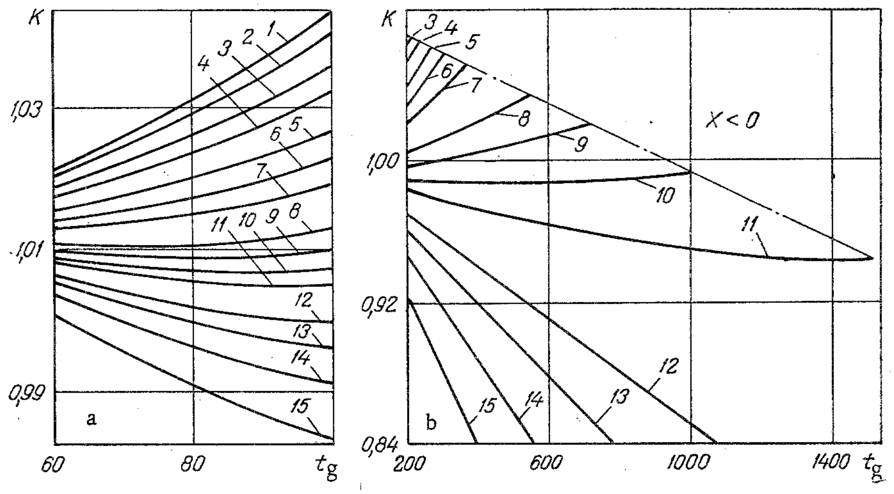

Fig. 1. Correction factor K as a functions of the gas temperature t_g, °C: (a) $t_g \leqslant 100$°C, (b) $t_g \geqslant 200$°C, at $t_\omega = 40$°C, and of the process beam parameter ε: 1) −50, 2) −100, 3) −200, 4) −300, 5) −500, 6) −700, 7) −1000, 8) −2000, 9) −3000, 10) −5000, 11) −10,000, 12) +10,000, 13) +5000, 14) +3000, 15) +2000.

$$t_g = \frac{t_g - 2500X_g}{c_g + c_v X_g} \tag{7}$$

for the temperature of the gas stream.

The enthalpy of the gas in the core of the stream can be expressed through the parameter ε characterizing the process beam

$$i_g = \varepsilon (X_g - X_\omega) + i_\omega . \tag{8}$$

Then

$$t_g = \frac{\varepsilon (X_g - X_\omega) + i_\omega - 2500X_g}{c_g + c_v X_g} . \tag{9}$$

Differentiation of Eq. (9) with respect to dX_g yields

$$\frac{dt_g}{dX_g} = \frac{c_g(\varepsilon - 2500) + c_v (\varepsilon X_\omega - i_\omega)}{(c_g - c_v X_g)^2} . \tag{10}$$

In calculations of dt_g/dX_g according to expression (10) it was assumed that $c_g = 1.0056$ J/kg·K and $c_v = 1.8855$ J/kg·K. Only in heat- and mass-transfer processes occurring at low temperatures of the gas stream $t_g < 100$°C (conditioner irrigation chambers, water cooling towers) can the specific heat of dry gas (air) and of water vapor be regarded as being constant. Meanwhile, scrubbers and dryers as well as contact-type economizers operate at temperatures of the gas stream $t_g = 100$–200°C [4], and contact-type water heaters operate at temperatures of the gas stream (products of combustion of natural gas) approaching 1500°C [5]. For this case the specific heat of each was determined according to the expressions $c_g = a + bt_g$ and $c_v = l + pt_g$, respectively, with $a = 0.980$, $b = 0.00013$, $l = 1.82$, and $p = 0.000408$ as the constants. For high temperatures of the gas stream we then have

$$t_g = \frac{a + l X_g}{2(b + pX_g)} + \sqrt{\left(\frac{a + l X_g}{2b + 2pX_g}\right)^2 - \frac{2500X_g - \varepsilon X_g + \varepsilon X_\omega - i_\omega}{b + pX_g}} , \tag{11}$$

and from here

$$\frac{dt_g}{dX_g} = \frac{bl - ap}{2(b + pX_g)^2} + \frac{0{,}5}{\sqrt{\left(\frac{a + l X_g}{2b + 2pX_g}\right)^2 - \frac{2500X_g - \varepsilon X_g + \varepsilon X_\omega - i_\omega}{b + pX_g}}} \times \left[\frac{2(bl - ap)}{(b + pX_g)^2} - \frac{(b + pX_g)(2500 - \varepsilon) - (2500X_g - \varepsilon X_g + \varepsilon X_\omega - i_\omega)}{(b + pX_g)^2}\right] . \tag{12}$$

On the basis of these relations, the values of the correction factor K have been determined for the following ranges of gas stream parameters: $-1500 \geqslant t_g \geqslant 40°C$, $10{,}000 \geqslant \varepsilon \geqslant 2000$, $-50 \geqslant \varepsilon \geqslant -10{,}000$, and the following three water temperatures: 20, 40, 60°C. These values were calculated according to expression (10) for low temperatures of the gas stream ($t_g \leqslant 100°C$) and according to expression (12) for high temperatures of the gas stream ($t_g \geqslant 200°C$). The results corresponding to the water temperature $t_w = 40°C$ are shown in Fig. 1.

At low temperatures of the gas stream (Fig. 1a) the correction factor varies from 0.983 to 1.043, its departure from unity increasing as the temperature of the gas stream rises and as the direction of the process beam departs from an X = const line ($\varepsilon \to \infty$). The trend is analogous at high temperatures of the gas temperature (Fig. 1b). The values of K for the process beam parameter equal to −50 and to −100 are not shown in Fig. 1b, because $X < 0$ within this range. The values of K for other negative values of the process beam parameter are limited for the same reason. For positive values of the process beam parameter the limiting value of K is 0.82, since values lower than that would correspond to a gas stream with a high vapor content and this condition is hardly ever encountered in industrial plants.

The values of factor K calculated for the water temperatures $t_w = 20$ and 60°C, respectively, indicate a slight decrease of this correction factor with higher water temperature. For temperatures of the gas stream $t_g > 500°C$ and a process beam parameter $\varepsilon = \infty$, however, the correction factor K decreases with higher water temperature. The variation of K with changes in the water temperature is not appreciable and does not exceed 3-4%.

From this discussion of the results one can conclude that it is permissible to disregard the correction factor in the analysis of heat and mass transfer at low temperatures of the gas stream ($t_g < 60°C$), as in air conditioners and in water cooling towers. For temperatures of the gas stream within the 100-200°C range, as in scrubbers and contact-type economizers, inclusion of the correction factor K depends on the direction of the process beam and on the required accuracy of analysis. Considering that contact-type water heaters operate at high temperatures of the gas stream, with the water temperature varying over a wide range in the direction of the process beam, one will find that here the correction factor must be included on account of its appreciable significance and that omitting it from calculations can results in large errors.

The results of this study can be used for a more accurate determination of the Lewis number in the analysis of heat and mass transfer in contact-type heat exchangers.

NOTATION

i, enthalpy; t, temperature; X, moisture content; c_g and c_v, specific heat of the gas and the vapor, respectively; r, heat of evaporation of the liquid; α_c, convective heat-transfer coefficient; β_x, moisture-transfer coefficient based on the difference of moisture contents; subscripts: g, gas; ω, liquid (water); and m, mean value.

LITERATURE CITED

1. O. Ya. Kokorin, Air Conditioning Plants [in Russian], Mashinostroenie, Moscow (1975).
2. L. D. Berman, Evaporation Cooling of Circulating Water [in Russian], Gosénergoizdat, Moscow—Leningrad (1957).
3. E. R. Eckert and R. M. Drake, Jr., Heat and Mass Transfer, McGraw-Hill (1959).
4. I. Z. Aronov, Contact Heating of Water by Products of Combustion of Natural Gas [in Russian], Nedra, Leningrad (1978).
5. Yu. P. Sosnin, Contact-Type Water Heaters [in Russian], Stroiizdat, Moscow (1974).